



## **Some reflections on the semantic approach, Tarskian truth and structuralism**

### **Algumas reflexões sobre a abordagem semântica, a verdade tarskiana e o estruturalismo**

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#### **Abstract**

In the present paper, we return to one of the main theses we have already defended, concerning the role of the Tarskian truth notion within the semantic approach (CARNIER, 2022). As it was argued, such a truth notion proves to be insufficient to be applied to scientific theories as they are conceived by this approach, i.e., as extralinguistic entities, because it is a property of sentences, and because the Tarskian truth of a sentence does not necessarily mean the world is as it describes. This implies that other truth conceptions, more appropriate, need to be articulated within the several members of the semanticist family, in order to characterize the relationship between theory and phenomenon. Our argument in this regard was based in a case study applied to constructive empiricism and quasi-realism, but in this paper, we extend our analysis to structuralism, assuming and endorsing the position according to which this proposal may be considered a member of the semantic approach.

#### **Keywords**

Tarskian truth, Semantic approach, Structuralism.

#### **Resumo**

No presente artigo, retomamos uma das principais teses que já defendemos, concernente ao papel da noção de verdade tarskiana no interior da abordagem semântica (CARNIER, 2022). Conforme argumentamos, ela se mostra insuficiente para ser aplicada às teorias científicas tal como concebidas por esta abordagem, isto é, como entidades extralinguísticas, por se configurar como uma propriedade de sentenças, e porque a

verdade tarskiana de uma sentença não significa necessariamente que o mundo é tal como ela o descreve. Isso implica no fato de que, outras concepções de verdade, mais apropriadas, devem ser articuladas no interior das diversas integrantes da família semanticista, para caracterizar a relação entre teoria e fenômeno. Nossa argumentação a esse respeito girou em torno de um estudo de caso que aplicamos ao empirismo construtivo e ao quase-realismo, porém no presente artigo, estenderemos nossa análise também ao estruturalismo, assumindo e endossando a posição segundo a qual esta proposta pode ser considerada uma integrante da abordagem semântica.

### **Palavras-chave**

Verdade tarskiana, Abordagem semântica, Estruturalismo.

## **1. Introduction**

As it was argued elsewhere (CARNIER, 2022), the truth notion developed by Alfred Tarski proves to be unnecessary and insufficient to be applied to scientific theories such as they are conceived within the semantic approach. The arguments leading us to conclude that it is unnecessary for this purpose are obtained from the reconstruction of scientific theories proportionated by the *partial structures approach*, which is a member of the semantic approach that allows us to conduct such reconstruction in light of an epistemic perspective, incorporating in the semantic analysis of theories certain characteristics of our knowledge, such as the fact that it may (and use to) be incomplete. With this, we can provide a more appropriate characterization of science both in terms of scientific practices and in terms of their main product, scientific theories<sup>1</sup>. However, in this paper we shall focus on the issue of the insufficiency of Tarski's truth notion to the semantic approach, and we will extend the analysis done in our previous paper about this point (CARNIER, 2022). But before we proceed, let us present the context in which our argument was established.

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<sup>1</sup> For more details we suggest our aforementioned paper, as well as other works which deal more deeply with the partial structures approach (cf. BUENO, 1999, cap. 4; BUENO, 2018 and DA COSTA; FRENCH, 2003).

## 1.1 Tarskian truth and model theory

The Tarskian truth notion can be seen as a formal counterpart of the *correspondence theory*, which says that a *sentence* is true if it corresponds to reality, and false otherwise. But either the truth or falsity of a sentence depend on how the expressions occurring in it are interpreted, and the Tarskian truth notion was developed to be applied in formalized languages; moreover, the element responsible for interpreting the expressions of the sentences of these languages is a *structure*. Hence, as far as formalized languages are concerned, its sentences can only be true or false (in the Tarskian sense) according to an interpretation in a structure.

These developments concerning the truth notion culminated in *model theory*, a branch of logic in which its semantic features are analyzed. According to Tarski, semantics is “a discipline which [...] *deals with certain relations between expressions of a language and the objects* (or ‘states of affairs’) *referred to’ by those expressions*” (TARSKI, 1949, p. 56; emphasis in original), so that in addition to the concept of structure, several others can be seen as semantic and will be explored in a formal context within model theory, including the Tarskian truth notion. Among these concepts lies the concept of *model*, from which the theory we are discussing gets its name. A model is a structure in which every sentence of a given set is true in the Tarskian sense<sup>2</sup>, and such dependency with respect to the Tarskian truth notion shows how this latter was important for model theory. One could even say that without it we would not have this theory, at least not with the same level of precision and rigour as we have today<sup>3</sup>.

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<sup>2</sup> Therefore, it is more appropriate to say *model of a set of sentences*. But loosely speaking we can simply say “model”.

<sup>3</sup> This might be seen, for example, when we read that the notion of truth in a structure already existed before Tarski, but was taken intuitively, without definition, thus generating some problems (cf. VAUGHT, 1974, p. 160-1). One of Tarski’s great merits was to define what is for a sentence to be true in a structure, what allowed him to define in the same lines other important semantic concepts. We can find the Tarskian truth notion/definition in several works, such as CHANG; KEISLER, 1990, p. 32 and specially in TARSKI; VAUGHT, 1957, p. 85, where it receives by the first time the standard formulation that it currently has in logic textbooks.

## 1.2 The semantic approach

The *semantic approach (view or conception) of theories* is a philosophical analysis of scientific theories which arises motivated mainly by the developments in formal logic discussed above, in such a way that for this proposal, the most important features of scientific theories are their semantic features in the model-theoretic sense. More specifically, for the semantic approach a theory can be identified with the class of its models, i.e., the class of structures in which its axioms are true, and since the Tarskian truth notion is essential to define this concept, it will be assumed by the semantic approach. But when we analyze its several members such as *constructive empiricism* and *quasi-realism*, we realize that this truth notion is not used when what is at stake is the truth or falsity of a scientific theory; on the contrary, what we see is the articulation of other conceptions which have a content quite different from that of Tarski, and prove to be more adequate to characterize what is supposed to be the proper relationship between theory and phenomenon.

We dealt in detail with the semantic approach in our previous paper (CARNIER, 2022). In this one, we shall make only some complementary remarks on the work of one of its pioneers, Patrick Suppes, and the main conception against which it opposes, *the received view of theories*.

The received view might be considered as the analysis of scientific theories underlying *logical positivism* (cf. SUPPE, 1974, p. 4), so that within this conception theories are construed as *first-order axiomatic systems partially interpreted*. Although the precise meaning of what it is for an axiomatic system to be partially interpreted has proved to be highly controversial (cf. PUTNAM, 1962), the fact is that the adherents of the received view establish a cleavage between *observable terms* and *theoretical terms* in the language of a theory (axiomatic system), only the former receiving a “direct” interpretation (being associated to observable entities or properties), while the latter are interpreted using the observable terms, by means of *correspondence rules*. Taking with some modifications an example from Rudolf Carnap (one of the proponents of the received view), a correspondence rule is a statement of the following kind, which relates the theoretical term “mass” with the observable relation “heavier than”: *if x is heavier than y, then the mass of x is greater than the mass of y* (cf. CARNAP, 1956, p. 48). Writing  $T$  to denote

the set of axioms of a given theory and  $C$  the set of its correspondence rules, on the received view the theory can be identified with the conjunction of  $T$  and  $C$ , and can be designated by  $TC$  (cf. SUPPE, 1974, p. 50-2).

The way that scientific theories are conceived within the received view gives an example of what Suppes calls *standard formalization* (cf. SUPPES, 2002, p. 24). In much of his work this author will focus on the question of the axiomatic reconstruction of physical theories, and the difficulty found in the way this is done through a standard formalization led him to propose a return to mathematics, more specifically to set-theory, with the aim of reconstruct them. According to Suppes, the most developed and relevant scientific theories usually require a recourse to mathematics, in such a way that the reconstruction of the former demands a simultaneous reconstruction of the latter, but when this is done by means of a standard formalization, the underlying logic is first order logic, which makes the final result unsatisfactory for several reasons, and in some cases the theories' reconstruction is even impossible (cf. SUPPES, 2002, p. 25-30). So Suppes proposes that mathematics be assumed in advance, by assuming set-theory, within which it can be developed, and physical theories be reconstructed from there, throughout a *set-theoretical predicate*, which is a predicate of (the language of) set-theory, defined using the axioms of the theory being reconstructed. What this predicate does is to determine the class of the models of the theory.

Taking one of Suppes' classic examples, a set-theoretical predicate for *group theory* is a predicate of the kind "is a group", defined as follows:

$\mathcal{G}$  is a group if and only if there exist  $G, \circ, e, ^{-1}$  such that

- (1)  $\mathcal{G} = \langle G, \circ, e, ^{-1} \rangle$
- (2)  $G$  is non-empty
- (3)  $\circ$  is a binary operation on  $G$
- (4)  $e$  is an element of  $G$
- (5)  $^{-1}$  is a unary operation on  $G$
- (6) for each  $x, y, z \in G$ :  $x \circ (y \circ z) = (x \circ y) \circ z$
- (7) for each  $x \in G$ :  $x \circ e = x$
- (8) for each  $x \in G$ :  $x \circ x^{-1} = e$

Thus, every structure that satisfies these eight axioms is a model of the theory and can be said a *group*.

From a logical point of view, Suppes' work also served as the basis for the emergence of a conception called *structuralism*, initially developed by his disciple Joseph D. Sneed, and afterwards by Wolfgang Stegmüller, C. Ulises Moulines and Wolfgang Balzer. We shall delve into this conception in section 2, for the time being we will make only the following remark. In his *The Semantic Conception of Theories and Scientific Realism*, Frederick Suppe rejects structuralism as a member of the semantic approach, on the basis of what he understands to be the use of certain resources of the received view, such as correspondence rules<sup>4</sup>, among other reasons. His criticisms received a reply by Pablo Lorenzano in the paper *The semantic conception and the structuralist view of theories: A critique of Suppe's criticisms*, where Lorenzano defends structuralism as an authentic member of the semanticist family (cf. LORENZANO, 2013). In the present paper we are assuming Lorenzano's stance, and we will endorse it with the conclusions that can be drawn from the thesis we defend.

To conclude this section, we will make an outline of the argument presented in our previous paper (CARNIER, 2022), which shows why the Tarskian truth notion can be considered insufficient to the semantic approach. Let  $\mathcal{M}$  be a model of a scientific theory  $\mathcal{T}$ , in which a hypothesis  $H$  like *All ravens are black*, and an initial condition  $I$  of the kind  *$c$  is a raven* (where  $c$  denotes a specific raven) are true. Note that  $\Gamma = \{H, I\}$  entails the prediction  $\alpha$  according to which  *$c$  is black*, and hence  $\alpha$  is true in  $\mathcal{M}$ . Since until now every raven we ever saw is indeed black, cases like this seem to suggest that the truth of a theory can be identified with the Tarskian truth of its statements in its models. But such an identification does not hold, as the following case shows.

Now let  $\mathcal{M}'$  be a model of a scientific theory  $\mathcal{T}'$ , in which a hypothesis  $H'$  like *all planets move in circular orbits*, which is one of the theory's axioms, and an initial condition  $I'$  of the kind

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<sup>4</sup> Thus leading him to state: “[...] one of the key distinguishing features of the Semantic Conception of Theories, as developed by Beth, van Fraassen, Suppes, and myself, is the absence of anything like correspondence rules [...]” (SUPPE, 1989, p. 20).

$p$  is a planet (where  $p$  denotes a specific planet), are true. Note that  $\Gamma' = \{H', I'\}$  entails the prediction  $\alpha'$  according to which  $p$  moves in a circular orbit, and hence  $\alpha'$  is true in  $\mathcal{M}'$ .  $\mathcal{T}'$  could be considered Copernicus' theory of planetary motion and  $\mathcal{M}'$  one of its models; nevertheless, the fact that  $\alpha'$  is true in  $\mathcal{M}'$  does not mean it holds, for as Kepler showed not long after Copernicus, the orbit of planets like Mars could not be circular, since it was elliptical (cf. KUHN, 1957, p. 211-3), so if  $p$  denotes the planet in question then  $\alpha'$  is false, whereas  $\neg\alpha'$  is true.

This case exemplifies what happens with the so-called “false” theories, and also shows that the Tarskian truth of a theory's statements in its models is insufficient to infer the truth of the theory itself, for the fact that a sentence is true in a structure does not imply the world is as the sentence describes it. Moreover, the Tarskian truth notion already proves to be inadequate to be applied to theories as they are conceived within the semantic approach, because it is a property of sentences, while for this approach theories are extralinguistic entities (cf. CARNIER, 2022 and DA COSTA; FRENCH, 2003, p. 22). All this will cause other truth conceptions to be developed within the members of the semanticist family; conceptions that, on the one hand, are more adequate to be applied to scientific theories as they are conceived within the semantic approach and, on the other hand, characterize more properly the relationship between theory and phenomenon.

Nonetheless, there is a problem with the way things were put, for in the latter case above we said that if  $p$  denotes Mars then  $\neg\alpha'$  is true, but a sentence can only be true or false in a structure and  $\neg\alpha'$  is false in  $\mathcal{M}'$ <sup>5</sup>. Thus, in which structure  $\neg\alpha'$  can be considered true? The answer to this question is closely related to the truth conceptions developed within the members of the semanticist family, so that in our previous paper (CARNIER, 2022) we saw how it is answered by constructive empiricism and quasi-realism, whereas in this work we shall see the answer given by structuralism.

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<sup>5</sup> This is an immediate consequence of Tarski's truth definition, since  $\alpha'$  is true in  $\mathcal{M}'$ .

## 2. Structuralism

One could say structuralism emerges with Joseph Sneed's *The Logical Structure of Mathematical Physics*. In this work Sneed focus on certain questions, like the question of the structure of scientific theories, the question of how this structure is employed to make empirical claims, the question of the theoretical and non-theoretical roles of a theory's concepts, and the question of how theories develop historically (cf. SNEED, 1979, p. VII). Sneed's ideas on these questions will mature and receive the contributions of Stegmüller, Balzer and Moulines, so that the final result of this process, which culminates in the official position of the structuralist program, is synthesized in the work *An Architectonic for Science*, from Sneed, Balzer and Moulines (1987).

In the present paper we shall restrict ourselves to the first three questions and to the most elementary notion of scientific theory we find within structuralism, the notion of *theory-element*, although many of our considerations also apply to the parts of this program that we will not deal with. A theory-element (or a theory, in this minimal sense) is a pair  $\mathbf{T} = \langle \mathbf{K}, \mathbf{I} \rangle$ , where  $\mathbf{K}$  is said a *theory-core* and  $\mathbf{I}$  is a set of *intended applications* of  $\mathbf{K}$  (cf. BALZER et al., 1987, p. 39). In its turn, the core  $\mathbf{K}$  of a theory-element is a 5-tuple  $\mathbf{K} = \langle \mathbf{M}_p, \mathbf{M}, \mathbf{M}_{pp}, \mathbf{GC}, \mathbf{GL} \rangle$  (cf. BALZER et al., 1987, p. 79). In what follows, we will analyze each of these components<sup>6</sup>.

The structuralists follow Suppes' approach and characterize the class of the models of the theory through set-theoretical predicates, which as we have seen, define the class of the structures in which the theory's axioms are true. However, these axioms can be divided into two kinds. On the one hand, we have those that, so to speak, state the "laws" of the theory and might be called *proper axioms*<sup>7</sup>. On the other hand, there are those determining the logical type

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<sup>6</sup> If  $\mathbf{T} = \langle \mathbf{K}, \mathbf{I} \rangle$  is a theory-element and  $\mathbf{K} = \langle \mathbf{M}, \mathbf{M}_p, \mathbf{M}_{pp}, \mathbf{GC}, \mathbf{GL} \rangle$  is a core, then  $\mathbf{K}$  can be denoted by  $\mathbf{K}(\mathbf{T})$ , and the factors that constitute it by  $\mathbf{M}_p(\mathbf{T})$ ,  $\mathbf{M}(\mathbf{T})$ ,  $\mathbf{M}_{pp}(\mathbf{T})$ ,  $\mathbf{GC}(\mathbf{T})$  and  $\mathbf{GL}(\mathbf{T})$ , respectively. From now on, when convenient, we shall use this notation.

<sup>7</sup> For the structuralists, a law is a formula that relates in a non-trivial way the different concepts of a theory. Taking an example from classical particle mechanics (CPM), a statement of the kind "force equals mass times acceleration" (Newton's second law), when properly formulated in an adequate language, can be considered as a proper axiom, since it establishes a non-evident connection between the concepts of *force* and *mass*, in addition to concepts such as *time* and *space* (in these last two cases the connection is established through the notion of *acceleration*). Hence,



of the structures candidates to be models of the theory. These latter are also known as *typifications* or *improper axioms*<sup>8</sup>, and the structures satisfying them are called *possible realizations* by Suppes (cf. DÍEZ; LORENZANO, 2002, p. 40), whilst the structuralists call them *potential models*. Given a theory  $\mathbf{T}$ ,  $\mathbf{M}_p(\mathbf{T})$  is the set of its potential models, and  $\mathbf{M}(\mathbf{T})$  is the set of its models, which are the structures satisfying the axioms of both classes. Thus every model is a potential model but not conversely, so  $\mathbf{M}(\mathbf{T}) \subseteq \mathbf{M}_p(\mathbf{T})$ .

The traditional distinction between theoretical and observable terms is rejected by the structuralists. For them, the most adequate distinction with regard to the reconstruction of scientific theories is between those concepts whose determination presupposes the validity of the theory's laws, hence the existence of a model in which they are true, and those concepts for which this is not so. Concepts of the first kind are called *T-theoretical*, while concepts of the second kind *T-non-theoretical* (cf. BALZER et al., 1987, p. 48-9). The determination of a *T-non-theoretical* concept is in charge of a theory  $\mathbf{T}'$  "preceding"  $\mathbf{T}$ . As far as metrical concepts (such as mass and force in mechanics) are concerned, which are usually formally characterized in terms of functions, their determination depends on *methods of measurement*, and the question

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Newton's second law is one of the laws which need to be satisfied by a structure, in order for it to be a model of **CPM**.

<sup>8</sup> Typifications inform the characteristics of the relations and operations that must constitute the structures candidates to be models of the theory. Taking **CPM** again as an example, if we consider a set  $P$ , an interval of real numbers  $T$ , two mappings  $m : P \rightarrow \mathbb{R}^+$  and  $s : P \times T \rightarrow \mathbb{R}^3$  (where  $s$  is twice differentiable on  $T$ ), and a mapping  $f : P \times T \times \mathbb{N} \rightarrow \mathbb{R}^3$ , then Newton's second law can be formulated as follows:

$$\text{For every } p \in P \text{ and } t \in T: m(p) \cdot d^2/dt^2 (s(p,t)) = \sum_{i \in \mathbb{N}} f(p, t, i).$$

In the reconstruction we are considering,  $P$  is a set of particles,  $T$  is a set of instants of time,  $m$  represents the notion of mass,  $s$  represents the notion of position (of a particle  $p$  at a given instant  $t$ ), whereas  $d^2/dt^2 s$  is the second derivative of  $s$  (hence it represents the notion of acceleration), and  $f$  represents the notion of force (acting on a particle  $p$  at an instant  $t$ ) (cf. DÍEZ; LORENZANO, 2002, p. 39-40). Restricting ourselves to a simple example, let us consider the notion of mass. Note that it is represented by a unary mapping  $m$ , which takes as argument a particle  $p$ , and assigns a real positive number  $m(p)$ , the numerical value of the mass of  $p$ . Therefore, the structures that do not contain a function of this kind are not models of the theory, given that for a structure to satisfy Newton's second law as formulated above, it must possess a unary mapping assigning to every particle  $p$  a positive real number. Thus, one of the typifications we find in the main reconstructions of **CPM**, states that the mapping  $m$  which constitutes the structures candidates to be models of the theory, maps their respective universes onto the set of positive real numbers (cf. DÍEZ; LORENZANO, 2002, p. 39; SUPPES, 1957, p. 294; MCKINSEY et al., 1953, p. 258; e BALZER et al., 1987, p. 103). The same holds, *mutatis mutandis*, for the mappings  $s$  and  $f$ .

of their theoreticity reduces to the question of whether *all* methods of measurement involved in their determination presuppose the laws of the theory (cf. BALZER et al., 1987, p. 50). If  $x$  is a potential model of a theory  $\mathbf{T}$ , the structure obtained by omitting  $x$ 's relations and functions that represent  $\mathbf{T}$ -theoretical concepts is said a *partial potential model*<sup>9</sup>, and  $\mathbf{M}_{pp}(\mathbf{T})$  is the set of the partial potential models of  $\mathbf{T}$ .

As we can see, the theoreticity of a concept is always relative to a theory  $\mathbf{T}$ , which means that a concept can be  $\mathbf{T}$ -non-theoretical but  $\mathbf{T}'$ -theoretical for another theory  $\mathbf{T}'$ <sup>10</sup>. We have already pointed out that the determination of a  $\mathbf{T}$ -non-theoretical concept is given by means of a theory that precedes  $\mathbf{T}$ . According to the structuralists this can be understood as a sort of information transference between theories; such transference can be formally characterized through the notion of *intertheoretic link*, which is a relation  $\mathbf{L}$  between potential models of the preceding theory  $\mathbf{T}'$ , and potential models of the preceded theory  $\mathbf{T}$ , i.e.,  $\mathbf{L} \subseteq \mathbf{M}_p(\mathbf{T}') \times \mathbf{M}_p(\mathbf{T})$  (cf. DÍEZ; LORENZANO, 2002, p. 71). Nevertheless, a theory  $\mathbf{T}$  may have links with several others<sup>11</sup>, so that the set of its potential models satisfying every link with other theories is said the *global link belonging to  $\mathbf{M}_p(\mathbf{T})$* , which is  $\mathbf{GL}(\mathbf{T})$  (cf. BALZER et al., 1987, p. 79). Since this is a set of potential models, we have that  $\mathbf{GL}(\mathbf{T}) \subseteq \mathbf{M}_p(\mathbf{T})$ .

In addition to links a theory also possesses *constraints*, which, to a certain extent, play a role similar to that of the axioms, in the sense that they also express certain characteristics of those things which satisfy them, except that in the case of constraints, the elements that shall satisfy them are not isolated structures, but combinations of potential models. In what concerns theories themselves, the function of the constraints can be better understood noting that the

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<sup>9</sup> We observe that the structure resulting from removing all of  $x$ 's relations and functions that represent  $\mathbf{T}$ -theoretical concepts is unique. This means we can establish a relationship between potential models and partial potential models of  $\mathbf{T}$ , throughout a mapping  $\mathbf{r} : \mathbf{M}_p(\mathbf{T}) \rightarrow \mathbf{M}_{pp}(\mathbf{T})$  which assigns to every  $y \in \mathbf{M}_p(\mathbf{T})$  the corresponding partial potential model.

<sup>10</sup> According to the structuralists, while the concept of position is not theoretical with respect to  $\mathbf{CPM}$ , it is theoretical with respect to *physical geometry* (cf. DÍEZ; LORENZANO, 2002, p. 71).

<sup>11</sup> Just like the concept of position, the concept of time is not theoretical with respect to  $\mathbf{CPM}$ , while it is theoretical with respect to *chronometry*. Therefore, in addition to physical geometry, classical mechanics also has links with this theory (cf. DÍEZ; LORENZANO, 2002, p. 71).

same theory may have several distinct applications which overlap. In those cases, some theories require that certain properties of objects appearing in such applications be preserved. Taking into account **CPM**, one of its constraints says that if  $p$  is a particle which occurs in two distinct mechanical systems (domains of application of the theory), then  $p$ 's mass is the same in both systems. This condition can be used to determine a class  $\mathbf{C} \subseteq \wp(\mathbf{M}_p(\mathbf{CPM}))$ , such that for each  $X \in \mathbf{C}$ , we have that: if  $x, y \in X$  and  $p \in P_x \cap P_y$  (where  $P_x$  and  $P_y$  are the domains of the potential models  $x$  and  $y$ , respectively), then  $m_x(p) = m_y(p)$  (where  $m_x$  is the mapping mass of  $x$  and  $m_y$  is the mapping mass of  $y$ ). The set  $\mathbf{C}$  is said the *equality constraint for mass in (CPM)*. This example allows us to glimpse the set-theoretical characteristics of the constraints of a theory  $\mathbf{T}$ , which are sets of combinations of potential models; in other words, they are subsets of  $\wp(\mathbf{M}_p(\mathbf{T}))$ . But as well as in the case of links,  $\mathbf{T}$  may have several constraints, and the set of every  $X \subseteq \mathbf{M}_p(\mathbf{T})$  satisfying them is said the *global constraint belonging to  $\mathbf{M}_p(\mathbf{T})$* , which is  $\mathbf{GC}(\mathbf{T})$ . Notice that  $\mathbf{GC}(\mathbf{T})$  is the intersection of all constraints for  $\mathbf{T}$ , therefore  $\mathbf{GC}(\mathbf{T}) \subseteq \wp(\mathbf{M}_p(\mathbf{T}))$ .

Summarizing, the core  $\mathbf{K}$  of a theory  $\mathbf{T}$  is composed by the set  $\mathbf{M}_p$  of its potential models (candidates to be actual models), the set  $\mathbf{M}$  of its models, the set  $\mathbf{M}_{pp}$  of its partial potential models (resulting from cutting  $\mathbf{T}$ -theoretical concepts in potential models), the set  $\mathbf{GC}$  which is the intersection of all of its constraints (which describe the relationships among  $\mathbf{T}$ 's distinct applications) and the set  $\mathbf{GL}$  containing  $\mathbf{T}$ 's potential models that satisfy its intertheoretical links (which describe the relationships between  $\mathbf{T}$ 's applications to those of other theories). And finally we get to the last component of a theory-element, the set  $\mathbf{I}$  of intended applications.

The intended applications of a theory represent those portions of reality for which the theory was developed to be applied. One of the most natural questions that arises in this regard is how this is determined, which leads to the question of how intended applications themselves are determined. According to the structuralists this process usually proceeds in two steps. First, the early proponents of the theory provide a set  $\mathbf{I}_0$  of *paradigmatic* cases. Note that what makes the elements of  $\mathbf{I}_0$  effectively intended applications is a decision of the theory's proponents. Lastly,  $\mathbf{I}_0$  is extended by adding all systems which have a satisfactory level of *similarity* to the initial systems, and thus one obtains  $\mathbf{I}$ .

Another important point about intended applications concerns the way they are formally characterized. According to the structuralists, the description of the domains to which a theory is intended to be applied is made through other theories that precede it (is made through theories with which the initial theory has links), by means of a previous vocabulary, i.e., by means of concepts non-theoretical with respect to it. Clearly, behind this conception of the relationship between theory and its domains of application is working the thesis of *theory-ladenness*. But these domains are not theory-laden by the theory used to account for them. They are theory-laden by the theories that precede the one used to account for them. Thus, intended applications may be conceived as partial potential models. Therefore, it follows that  $\mathbf{I} \subseteq \mathbf{M}_{pp}$ .

Given a core  $\mathbf{K} = \langle \mathbf{M}, \mathbf{M}_p, \mathbf{M}_{pp}, \mathbf{GC}, \mathbf{GL} \rangle$  of a theory  $\mathbf{T}$ , it is possible to obtain from the sets  $\mathbf{M}$  and  $\mathbf{GL}$  together with  $\mathbf{GC}$ , a specific selection of combinations of potential models.  $\wp(\mathbf{M})$  provides combinations of  $\mathbf{T}$ 's models, whereas  $\wp(\mathbf{GL})$  provides combinations of potential models that satisfy all of  $\mathbf{T}$ 's links to other theories. Finally,  $\wp(\mathbf{M}) \cap \mathbf{GC} \cap \wp(\mathbf{GL})$  provides combinations of potential models which besides satisfy the global constraint, possess as elements only those models of  $\mathbf{T}$  that satisfy the global link.  $\wp(\mathbf{M}) \cap \mathbf{GC} \cap \wp(\mathbf{GL})$  is said the *theoretical content* of  $\mathbf{K}$ , and is denoted by  $\mathbf{Cn}_{th}(\mathbf{K})$  (cf. BALZER et al., 1987, p. 82). We observe that (1)  $\mathbf{Cn}_{th}(\mathbf{K}) \subseteq \wp(\mathbf{M}_p)$ ; (2) the mapping  $\mathbf{r}$  can be extended to the power sets of  $\mathbf{M}_p$  and  $\mathbf{M}_{pp}$ , so as to obtain a mapping  $\mathbf{r}_\wp : \wp(\mathbf{M}_p) \rightarrow \wp(\mathbf{M}_{pp})$ , such that for each  $X \subseteq \mathbf{M}_p$ ,  $\mathbf{r}_\wp(X) = \{\mathbf{r}(x) \in \mathbf{M}_{pp} : x \in X\}$ ; and (3) the mapping  $\mathbf{r}$  can be restricted to  $\mathbf{Cn}_{th}(\mathbf{K})$ , in such a way that the range of this restriction, in symbols  $\text{Rng}(\mathbf{r}_\wp \upharpoonright \mathbf{Cn}_{th}(\mathbf{K}))$ , is composed by combinations  $Y$  of partial potential models, such that  $Y$ 's elements can be "extended" by adding relations and functions which represent  $\mathbf{T}$ -theoretical concepts, so as to become models of  $\mathbf{T}$  that satisfy the global link. Furthermore, the set  $X$  of all these extensions satisfies the global constraint.  $\text{Rng}(\mathbf{r}_\wp \upharpoonright \mathbf{Cn}_{th}(\mathbf{K}))$  is said the *content* of  $\mathbf{K}$ , and is denoted by  $\mathbf{Cn}(\mathbf{K})$ <sup>12</sup>.

<sup>12</sup>  $\wp(\mathbf{M}) \cap \mathbf{GC} \cap \wp(\mathbf{GL})$  can also be called the *theoretical content* of  $\mathbf{T}$ , and is denoted by  $\mathbf{Cn}_{th}(\mathbf{T})$ , while  $\text{Rng}(\mathbf{r}_\wp \upharpoonright \mathbf{Cn}_{th}(\mathbf{K}))$  can also be called the *content* of  $\mathbf{T}$ , and is denoted by  $\mathbf{Cn}(\mathbf{T})$ . So, we have that  $\mathbf{Cn}_{th}(\mathbf{K}) = \mathbf{Cn}_{th}(\mathbf{T})$  and  $\mathbf{Cn}(\mathbf{K}) = \mathbf{Cn}(\mathbf{T})$  (cf. BALZER et al., 1987, p. 90). The way we defined  $\mathbf{Cn}(\mathbf{K})$  is slightly different from that we find in the work *An Architectonic for Science* (cf. BALZER et al., 1987, p. 85). However, it is easy to check that both definitions give us the same set.

According to the structuralists, a theory-element might be considered as a tool to formulate *empirical claims*. Indeed, each theory-element  $\mathbf{T} = \langle \mathbf{K}, \mathbf{I} \rangle$  is associated with an empirical claim, according to which the set  $\mathbf{I}$  of intended applications belongs to the content of  $\mathbf{K}$ . Formally,  $\mathbf{I} \in \mathbf{Cn}(\mathbf{K})$  (cf. DÍEZ; LORENZANO, 2002, p. 66). In other words, the empirical claim of  $\mathbf{T}$  states that its intended applications are partial potential models susceptible of being “extended” through the addition of relations and functions that represent  $\mathbf{T}$ -theoretical concepts, so as to become models of  $\mathbf{T}$  that satisfy the global constraint. If this actually happens, the theory can be successfully applied to those domains for which it was developed to be applied.

The empirical claim of a theory  $\mathbf{T} = \langle \mathbf{K}, \mathbf{I} \rangle$  can be used to establish a derivative truth notion.  $\mathbf{T}$  can be said true if its empirical claim is true, i.e., if it is the case that  $\mathbf{I} \in \mathbf{Cn}(\mathbf{K})$ . The idea of reducing the truth of a theory to the truth of its empirical claim was suggested by a disciple of Suppes named Ernest Adams (cf. DÍEZ; LORENZANO, 2002, p. 44 and ADAMS, 1959, p. 260); although Adams’ views on scientific theories are simpler and considerably different from those of the structuralists. More precisely, Adams conceives a theory as an ordered-pair  $\langle C, I \rangle$ , where  $C$  is called its *characteristic set* (which is the set of its models), and  $I$  is said its *set of intended models* (whose elements are those structures that represent the domains for which the theory was developed to be applied) (cf. ADAMS, 1959, p. 259). Moreover, within Adams’ program the structures of these latter set might be models of the theory, and this is precisely what the empirical claim of the theory says, that its intended models are actually its models, i.e.,  $I \subseteq C$  (cf. ADAMS, 1959, p. 260). Adams’ work is previous to structuralism, and was one of the motivations for the development of this latter from a philosophical point of view, mainly because Adams’ program presented some difficulties (cf. DÍEZ; LORENZANO, 2002, p. 44). In any case, such as formulated within structuralism, the empirical claims associated with each theory do not merely state that the intended applications belong to its content.

In model-theoretic terms, the potential models and the partial potential models of a theory are related specifically. Taking into account **CPM** once more, its potential models are structures of the kind  $x = \langle P, T, s, m, f \rangle$  (cf. DÍEZ; LORENZANO, 2002, p. 39, 60-1), whilst its partial

potential models are structures of the kind  $y = \langle P, T, s \rangle$  (cf. DÍEZ; LORENZANO, 2002, p. 63-4)<sup>13</sup>. Note that both  $x$  and  $y$  have the same base-sets ( $P$  and  $T$ ), and all functions occurring in  $y$  also occur in  $x$ . When this happens,  $y$  is said a *reduct* of  $x$  and  $x$  is said an *expansion* of  $y$ <sup>14</sup>. So, the empirical claim of a theory states implicitly that its intended applications are reducts of those models that satisfy all links, and the sets which contain these models satisfy the global constraint. Therefore, behind the truth notion underlying structuralism are working several relations between structures, especially the reduct relation between the intended applications of a theory and its models, as well as the relations of the theory's models that satisfy the global link to the potential models of other theories.

Returning to the hypothetical case at the end of section 1.2, we could say that the negation of prediction  $\alpha'$  ( $\neg\alpha'$ ) is true in an intended application  $\mathcal{F}$  of theory  $\mathcal{T}'$ , and so  $\mathbf{I} \notin \mathbf{Cn}(\mathcal{T}')$ , for since  $\alpha'$  is true in  $\mathcal{M}' \in \mathbf{M}(\mathcal{T}')$ , we have that  $\mathcal{F}$  is not a reduct of  $\mathcal{M}'$  nor of any other structure  $x \in \mathbf{M}(\mathcal{T}')$ ; otherwise  $\neg\alpha'$  would be true in  $x$  and  $H'$  would be false, which is absurd given that  $H'$  is one of the axioms of  $\mathcal{T}'$ . To put it another way, there exists at least one intended application that cannot be extended to a model of the theory. So for structuralism, although a prediction may be true in a model of a theory, if its negation is true in some intended application, the empirical claim associated with the theory is false and hence the theory itself may be said false.

### 3. Conclusion

The considerations made above corroborate and expand what we defended in our previous paper (CARNIER, 2022), this time showing how the structuralist program deals with the issue of the insufficiency of Tarski's truth notion. The same way as in the case of the other members of the semanticist family, this is done by developing an alternative truth conception that depends essentially on relationships between structures, and conforms to the theoretical constraints of the program where it is developed. As we stated (CARNIER, 2022), this points to a natural tendency of the semantic approach, which consists of making certain notions such as

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<sup>13</sup> The meaning of each of  $x$ 's components can be found in note 8 above.

<sup>14</sup> For more details on the notion of reduct, cf. CHANG; KEISLER, 1990, p. 20.

the truth notion “more semantic”, insofar as they cease to be properties of syntactic entities like sentences and become properties of theories conceived as fundamentally semantic entities (like structures, classes of structures, etc.). This is true of constructive empiricism and quasi-realism as well as structuralism, so that, regardless of whether this last proposal is a member of the semantic approach (which we think is the case!) or not, it certainly shares some of its semanticist aspirations with the first two.

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