revista observatório

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12, 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a25en

K-SUN BELONGS TO HELLY B2-EPG

K-SUN PERTENCE A B2-EPG-HELLY
K-SUN PERTENECE A B2-EPG-HELLY

Tanilson Dias dos Santos
Doctor in Systems and Computing Engineering - PESC/COPPE-UFRJ. Professor of the Computer Science Course - Federal University of Tocantins, Brazil. tanilson.dias@mail.uft.edu.br.
0000-0002-0636-5751

Kedson Alves Silva
Bachelor of Computer Science – Federal University of Tocantins. Brazil. alves.kedson@mail.uft.edu.br.
0000-0003-0370-3118

Luis Fernando dos Santos Marinho
Bachelor of Computer Science – Federal University of Tocantins. Brazil. fernando.marinho@mail.uft.edu.br.
0000-0003-0412-7431

Mailing address: Universidade Federal do Tocantins, Reitoria, DIRETORIA DE COMUNICAÇÃO. Quadra 109 Norte Avenida NS 15, Plano Diretor Norte, 77001090 - Palmas, TO – Brasil

Received: 03.14.2023.
Accepted: 05.16.2023.
Published: 06.02.2023.

ABSTRACT:
In this article we explore the B_2-EPG class and the Helly property. We present generic results on EPG representations and define terms that support the other results, in addition, we finish the research with an unpublished algorithm that builds a Helly B_2-EPG representation of any *k*-sun graph.

KEYWORDS: B_2-EPG; Graph Theory; Intersection Graphs; Helly Property; *k*-sun Graphs.

Introduction

EPG is the acronym of Edge-intersection Graphs of Paths on a Grid. They correspond exactly to graph class of path intersection on a grid. The EPG graph class was defined by Golumbic, Lipshteyn and Stern (2009), studied later by Golumbic, Lipshteyn and Stern (2013) in addition to being explored by recent works in the literature (Asinowski; Suk, 2009; Cameron; Chaplick; Hoàng, 2013; Heldt; Knauer; Ueckerdt, 2014a; Szwarcfiter et al., 2020; Alcón; Mazzoleni; Santos, 2021; Marinho; Silva; Santos, 2023). In particular, the class of B_2-EPG graphs has been less studied, where we can cite the work of Heldt, Knauer and Ueckerdt (2014b) who addressed the bend number of planar and outerplanar graphs, Biedl and Stern (2010) who classified line graphs and bipartite planar graphs as B_2-EPG, as well as Alcón et al. (2018) and Francis and Lahiri (2016) which respectively classified the normal circlearc graphs and the Halin graphs as B_2-EPG. Among the most important works we can mention Pergel and Rzążewski (2017) that presents an NP-completeness proof for the B_2-EPG graphs recognizing problem.

An *EPG representation* of a graph G, denoted by R_G, is an edge-intersection model of paths on a grid. There is a bijective relationship among the set of vertices and edges of the graph G and its corresponding EPG representation. We denote each vertex $v_i \in V(G)$ as a related path P_{v_i} drawn on the grid, and two vertices V_i, V_j are adjacent to each

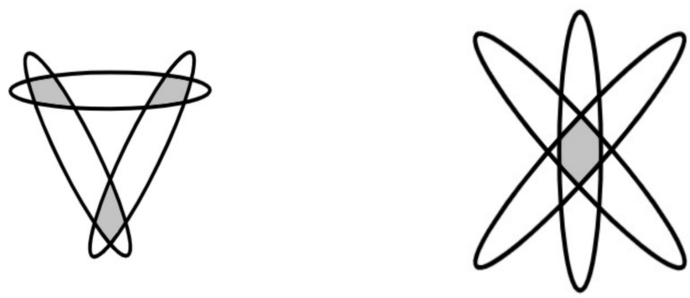
e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

other if and only if the corresponding paths (P_{v_i} and P_{v_j}) intersect in at least one edge. We denote each path of the representation as a set of edges (and not of vertices), so that the path is considered as a finite sequence of consecutive edges $e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3)$, $e_3 = (v_3, v_4)$, ..., $e_i = (v_i, v_{i+1})$, ..., $e_m = (v_m, v_{m+1})$, where $v_i \neq v_j$ for $i \neq j$. A *path in the representation* is a path hosted on the grid.

A bend is defined as a pair of consecutive edges, e_1 and e_2, that have different directions on the grid. When this happens e_1 and e_2 are called bend edges. If a path has no bends it is called a segment. Still on EPG representations, we say that it is a B_k-EPG representation, when all paths of the representation have at most k bends, i.e. k changes of direction, where $k \geq 0$ and k is an integer.

A very famous property of the *Set Theory* is the *Helly* property. This property of intersecting sets can be defined as follows: A collection of sets satisfies the Helly property when every subcollection that is mutually intersecting has at least one element in common. In an EPG representation of a graph G, we observe the Helly property with respect to edge-intersections of paths. When a collection of paths is mutually intersecting, if this collection of paths shares at least one common edge of the representation then it is a Helly representation.

Figura 1 Sets that do not satisfy the Helly property (at left) and sets that satisfy the property (at right), respectively



e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

Figura 2 A graph G (at left), a non-Helly B_2-EPG representation of G (in the center), and a Helly B_2-EPG representation of G (at right), respectively

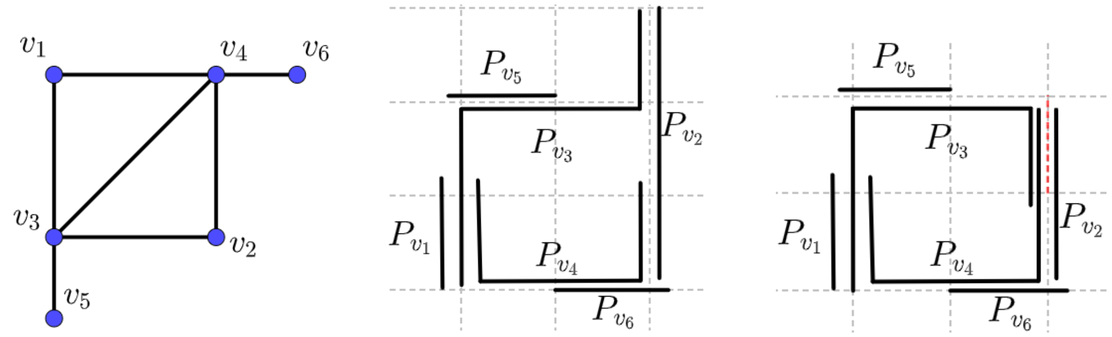


Figure 1 presents two families of subsets, each family of subsets is pairwise intersecting with each other, i.e. is a 2 to 2 intersecting family, or we can say that any two elements of this family have non-empty intersection. The family of sets depicted in Figure 1, at the left, is pairwise intersecting but the intersection of all the sets is empty, so it is an example of a family of sets that does not satisfy the Helly property. On the other hand, depicted in Figure 1, at the right, we have a collection of pairwise intersecting sets whose intersection of all elements is not empty, so this collection satisfies the Helly property.

We can observe the Helly property in Graph Theory. In particular, in EPG graphs we observe the Helly property of the intersections of the set of paths in the representation. In Figure 2 is depicted a graph G, at the left, with two of its possible B_2-EPG representations. In Figure 2, at the right, we have a representation where the paths P_{v_2}, P_{v_3}, P_{v_4} intersect and they have a common edge, so the representation is Helly. On the other hand, in Figure 2, at the center, we have a set of pairwise intersecting paths but there is no edge of the common representation, simultaneously, to these 3 paths, thus constituting a non-Helly representation.

Methodology

In this work we adopted an exploratory and investigative research methodology. It is investigative and exploratory because its goal is to investigate or to know something that, through this research, gives a more complete point of view about the problem under study. In this work we seek to explore, in general, the B_2-EPG graph class and the related Helly property.

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

This research investigate the B_2-EPG graphs and generic results about relationships among paths. The following section brings some definitions and general results of the work, in addition we present algorithms that generate B_2-EPG representations of some particular graphs.

Some Results for B_2-EPG Graphs and Others

Follow we present some of the results obtained from the study of the B_2-EPG class.

a. Generic Results

Lemma 1. Let L_1 and L_2 be two grid lines parallel to each other, if a path P_1 edge-intersects L_1 and L_2 then the path P_1 necessarily has at least two bends.

Proof. Without loss of generality, consider any two distinct horizontal (thus parallel) lines of the grid, say L_1 and L_2. If path P_1 has a segment on line L_1, then it must make at least one bend to reach line L_2. However, if P_1 reaches L_1 and does not bend again then P_1, at most, will vertex-intersect Line L_2. Therefore, for there to be a segment of P_1 on L_2, P_1 must have at least two bends.

#

Corollary 2. If the path $P_1 \in B_2$-EPG, where P_1 has two bends, then P_1 has exactly two segments in the same direction (either horizontal or vertical).

Proof. In a rectangular grid there are only two directions (horizontal and vertical). If path P_1 has two bends, it is necessarily composed of three segments. Therefore, by the pigeonhole principle, we need to accommodate these three segments in the two possible directions, i.e., we have more segments than directions to accommodate. Inevitably we will have to accommodate two segments in one of the directions.

#

Definition 3. Let P_1 be a path B_2-EPG with two bends. We call the central segment of P_1 the body, while the segments attached to the body are called the legs.

Corollary 4. Let $P_1 \in B_2$-EPG, where P_1 has two bends and three segments. The body segment of P_1 has the opposite direction to the leg segments of P_1.

Proof. By Corollary 2 we know that there are two segments in the same direction. Assume by contradiction that the body and at least one of the legs have the same direction. Now, but if that happens we will have two consecutive segments of the same path in the same direction. In this case, either the segments are on the same grid line, if

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

they are together, one at the end of the other, they actually form the same segment. Or they are on the same line and there is a discontinuity, so it does not form a path. Or they are on different grid lines, in which case by Lemma 1 the path has more than two bends. In all cases we concluded a contradiction.

#

Corollary 5. Given $P_1 \in B_2$*-EPG where* P_1 *has two bends and three segments, if* P_1 *has two segments in the same direction, then these two segments are on parallel lines of the representation grid.*

Proof. By Corollary 4, we know that the body of P_1 has the opposite direction to the direction of the legs, in addition, at each extremity of the segment of the body of P_1 there is a bend for the segments of the legs. Let is consider P_1 formed by the segments S_1, S_2 and S_3, with S_2 being the body, and considering any line of the representation grid, say L_1, if S_2 is on L_1 from column C_1 to column C_2, as S_1 and S_3 must have direction opposite to S_2 and share 1 extremity vertex with S_2, then S_1 and S_3 must necessarily be one on C_1 and the other on C_2. Since C_1 and C_2 are columns at the extremities of segment S_1, we conclude that S_1 and S_3 are therefore parallel to each other.

#

Lemma 6. Let S_1 *and* S_2 *be two distinct path segments of an EPG representation, if* $S_1 \cap_v S_2 \neq \emptyset$ *and* $S_1 \cap_e S_2 = \emptyset$*, then at most* $|S_1 \cap_v S_2| = 1$.

Proof. In an EPG grid representation, a path segment lies solely on a grid line. So let is say that S_1 and S_2 are on the grid lines L_1 and L_2 respectively, where L_1 and L_2 can be lines of different directions, lines in the same direction but parallel or they can be the same line.

- If L_1 and L_2 are lines in the same direction but parallel, then $|S_1 \cap_v S_2| = \emptyset$;
- If L_1 and L_2 are lines of different directions, then L_1 and L_2 can intersect at exactly one point, so at most $|S_1 \cap_v S_2| = 1$;
- If L_1 and L_2 are the same line, for S_1 and S_2 to share more than one vertex then they must be consecutive vertices, since they are on the same line. In the representation grid, two consecutive vertices form an edge, that way, if S_1 and S_2 share more than one consecutive vertex, then $S_1 \cap_e S_2 = \emptyset$, so at most $|S_1 \cap_v S_2| = 1$;

Given these facts, we conclude that at most $|S_1 \cap_v S_2| = 1$.

#

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

Lemma 7. Given a path $P_1 \in B_2$-EPG where P_1 has two bends, and let S_2 be a segment of the same representation. If $|P_1 \cap_v S_2| \neq \emptyset$ and $|P_1 \cap_e S_2| = \emptyset$, then at most $|P_1 \cap_v S_2| = 2$.

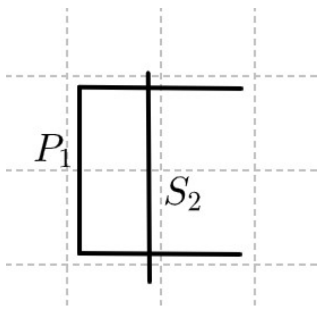
Proof. By Corollary 4 we know that the body of P_1 has the opposite direction to the other segments. By Corollary 5, P_1 is legs are not on the same grid line. By Lemma 6, we know that two segments that do not share an edge can vertex-intersect in at most one point. By contradiction, suppose that $|P_1 \cap_v S_2| > 2$, say $|P_1 \cap_v S_2| = 3$, so there must be a configuration where S_2 vertex-intersects the three segments of P_1. By Lemma 6 we know that two segments that do not intersect on an edge can have at most one vertex in common. So for S_2 to have three points of intersection with path P_1, necessarily S_2 would have to intersect the three segments of P_1.

- S_2 cannot be on grid lines parallel to P_1 is legs, since if positioned that way S_2 could vertex-intersect only one of the three segments of P_1 or a bend vertex of P_1, so $|P_1 \cap_v S_2| = 1$.
- Case S_2 is on a grid line perpendicular to the lines where the two legs of P_1 are positioned. If this line is the same as P_1 is body is drawn, then S_2 could vertex-intersect P_1 only at one extremities of its body, so $|P_1 \cap_v S_2| = 1$. If S_2 is hosted on a line that allows it to vertex-intersect the legs of P_1:
  - If the legs bend to different sides: in this case $|P_1 \cap_v S_2| = 1$;
  - If both legs bend to the same side: with this configuration $|P_1 \cap_v S_2| = 2$, see Figure 3.

Therefore, we conclude in a contradiction because in all tested configurations, at most $|P_1 \cap_v S_2| = 2$.

#

Figura 3 Intersection of a segment with a B_2-EPG path



e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

b. k-sun Graph

Definition 8. A k-sun graph S_k, with k ≥ 3, consists of 2k vertices, where there is an independent set $X = \{x_1, \dots, x_k\}$ a clique $Y = \{y_1, \dots, y_k\}$ and edges $E_1 \cup E_2$, where $E_1 = \{x_1y_1, y_1x_2, x_2y_2, y_2x_3, \dots, x_ky_k, y_kx_1\}$ form the external cycle and $E_2 = \{y_iy_j | i \neq j\}$ form the inner clique.

Definition 9. The mod operator is used to return the remainder of integer division. When the division is exact, it returns zero. When dividend is less than divisor, then returns the dividend itself. Examples: 4 mod 8 = 4; 12 mod 4 = 0; 10 mod 4 =2; 5 mod 4=1.

According to results presented by Golumbic, Lipshteyn and Stern (2009), *k*-sun ∉ B_1-EPG, for $k \geq 4$. Next, we prove that *k*-sun ∈ B_2-EPG for all $k \geq 3$. After the first feedback about a work submitted to the CSBC ETC congress, we were surprised by the fact that the result that appears in our Lemma 10 would have already been presented, in the literature, in the paper by Cela and Gaar (2019), where there is a result similar to that of this work, however, with a different approach. The distinct approach allows us to conclude the Corollary 11.

Lemma 10. The k-sun graph ∈ B_2 -EPG.

Proof. Consider row l_0 and column c_0 as the central row and column of the representation, see Figure 2. We consider two particular cases in our construction:

- *k* even: The path P_{y_i} to $i = 1$, in this case P_{y_i} has a horizontal segment on line l_0 between columns c_{-1} and $c_{\frac{k}{2}}$. In column c_{-1}, the path P_{y_i} bends and extends to line l_1, whereas in column $c_{\frac{k}{2}}$, if k mod 4 = 0, then P_{y_i} bends and extends to line l_{-1}, otherwise P_{y_i} bends and extends to line l_1.
- *k* odd: The path P_{y_i} for $i = 1$, has a horizontal segment on the line l_0 between columns c_{-1} and c_0. In column c_{-1}, the path P_{y_i} bends and extends to line l_1, whereas in column c_0, P_{y_i} bends and extends to line l_{-1}. The path P_{y_i} for $i = k$, has a horizontal segment on line l_0 between columns c_0 and $c_{\frac{k-1}{2}}$. If (k+1) mod 4 ≠ 0, both extremities of the path P_{y_i}, respectively on columns c_0 and $c_{\frac{k-1}{2}}$ bend and extend to line l_{-1}, otherwise the path P_{y_i} bends in c_0 and extends to line l_{-1}, and also bends at $c_{\frac{k-1}{2}}$ and extends to line l_1. The path P_{x_i}, for $i = k$, is represented by a vertical segment on column c_0 between lines l_0 and l_{-1}.

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

To represent the vertices of the clique Y follow the procedure below. The algorithm represents each path P_{y_i}, where i is odd. When k is even, consider $i \neq 1$, and when k is odd, consider $i \neq 1$ and $i \neq k$, as these paths were previously constructed by the algorithm. In this case we have two subcases, case 1.1, when $(i+1)$ mod 4 = 0, then P_{y_i} has a horizontal segment on line l_0, between columns $c_{-\frac{i+1}{2}}$ and $c_{\frac{i-1}{2}}$. In column $c_{-\frac{i+1}{2}}$, the path P_{y_i} bends and extends to line l_{-1}, while in column $c_{\frac{i-1}{2}}$, P_{y_i} bends and extends to line l_1; case 1.2, when $(i+1)$ mod 4 ≠ 0, then P_{y_i} has a horizontal segment on line l_0, between columns $c_{-\frac{i+1}{2}}$ and $c_{\frac{i-1}{2}}$. In column $c_{-\frac{i+1}{2}}$, the path P_{y_i} bends and extends to line l_1, while in column $c_{\frac{i-1}{2}}$, P_{y_i} bends and extends to line l_{-1}.

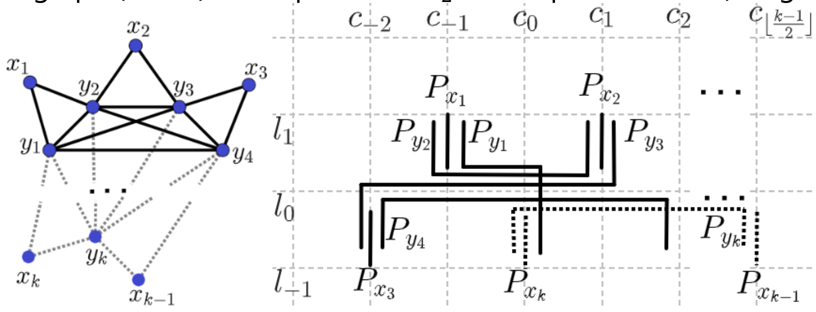
Let P_{y_i} be the path, where i is even and i mod 4 ≠ 0. In this case, P_{y_i} has a horizontal segment on line l_0 between columns $c_{-\frac{i}{2}}$ and $c_{\frac{i}{2}}$. Both extremities of the P_{y_i} path bend, respectively in columns $c_{-\frac{i}{2}}$ and $c_{\frac{i}{2}}$, and extend to line l_1. Case path P_{y_i} when i is even and i mod 4 = 0, then P_{y_i} has a horizontal segment on line l_0 between columns $c_{-\frac{i}{2}}$ and $c_{\frac{i}{2}}$. Both extremities of path P_{y_i}, respectively in columns $c_{-\frac{i}{2}}$ and $c_{\frac{i}{2}}$ bend and extend to line l_{-1}.

The vertices of the independent set X are represented by vertical segments that occupy different columns. Consider $i \neq$ k when k is odd. In the case of path P_{x_i}, where i is even and i mod 4 = 0, P_{x_i} is represented by a segment on column $c_{\frac{i}{2}}$ between lines l_0 and l_{-1}. For P_{x_i}, where i is even and i mod 4 ≠ 0, P_{x_i} is represented by a segment on column $c_{\frac{i}{2}}$ between lines l_0 and l_1. The path P_{x_i}, where i is odd and $(i+1)$ mod 4 = 0, P_{x_i} is represented by a segment on column $c_{-\frac{i+1}{2}}$ between lines l_0 and l_{-1}. For P_{x_i}, where i is odd and $(i +1)$ mod 4 ≠ 0, P_{x_i} is represented by a segment on column $c_{-\frac{i+1}{2}}$ between lines l_0 and l_1.

#

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

Figura 4 k-sun graph (at left) and a particular B_2-EPG representation (at right)



To facilitate understanding, we write an algorithm to describe the lemma 10, the code can be found in Algorithm 1, in page 19.

Corollary 11. The k-sun graph $\in$ Helly B_2-EPG.

Proof. Consider row l_0 and column c_0 as the central row and column of the representation. We consider in our construction three particular cases:

- k even: The path P_{y_i}, for i = k−1, has a horizontal segment on line l_0 between columns c_0 and $c_{\lfloor\frac{k-1}{2}\rfloor}$. In column c_0, P_{y_i} bends and extends to line l_1, while in column $c_{\lfloor\frac{k-1}{2}\rfloor}$ P_{y_i} bends and extends to line l_{-1}. The path P_{y_i}, for i = k, has a vertical segment on column c_0 between lines l_0 and l_1. In line l_0, P_{y_i} bends and extends to column c_1, whereas in line l_1, P_{y_i} bends and extends to column $c_{-\lfloor\frac{k-1}{2}\rfloor}$. The P_{x_i} path, for i = k −1 is represented by a vertical segment on column c_0 between lines l_0 and l_1.
- k odd and k ≠ 3: The path P_{y_i}, for i = k – 1, has a horizontal segment on the line l_0 between columns $c_{-(\lfloor\frac{k-1}{2}\rfloor-1)}$ and $c_{\lfloor\frac{k-1}{2}\rfloor}$. In column $c_{-(\lfloor\frac{k-1}{2}\rfloor-1)}$, P_{y_i} bends and extends to line l_{-1}. The path P_{y_i}, for i = k has a vertical segment on column c_0 between lines l_0 and l_1. In line l_0, P_{y_i} bends and extends to column $c_{\lfloor\frac{k-1}{2}\rfloor}$, whereas in line l_1, P_{y_i} bends and extends to column $c_{-\lfloor\frac{k-1}{2}\rfloor}$. The path P_{x_i}, for i = k −1 is represented by a horizontal segment on line l_0 between columns $c_{\lfloor\frac{k-1}{2}\rfloor-1}$ and $c_{\lfloor\frac{k-1}{2}\rfloor}$.
- k = 3: The path P_{y_i}, for i = k −1 = 2, is represented by a horizontal segment on line l_0 between columns c_{-1} and c_2. The path P_{y_i}, for i = k = 3 has a vertical segment on column c_0 between lines l_0 and l_1. In line l_0, P_{y_i} bends and extends to

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

column c_2, while in line l_1, P_{y_i} bends and extends to column c_{-1}. The path P_{x_i}, for $i = k - 1 = 2$ is represented by a horizontal segment on the line l_0 between columns c_1 and c_2.

To represent the vertices of the clique Y follow the procedures below. During the execution of the following steps if $k = 3$ or $k = 4$, consider $-(\left\lfloor\frac{k-1}{2}\right\rfloor - 1) = 0$. The path P_{y_i}, for $i = 1$, has a vertical segment on the column $c_{-\lfloor\frac{k-1}{2}\rfloor}$ between lines l_0 and l_1. In line l_1 P_{y_i} bends and extends to column $c_{-(\left\lfloor\frac{k-1}{2}\right\rfloor-1)}$, whereas in line l_0, P_{y_i} bends and extends to column c_1. The path P_{y_i}, for $i = 2$ (except when $k = 3$, as it was already defined earlier), has a horizontal segment on line l_0 between columns $c_{-\lfloor\frac{k-1}{2}\rfloor}$ and c_1. In column c_1 P_{y_i} bends and extends to line l_{-1}.

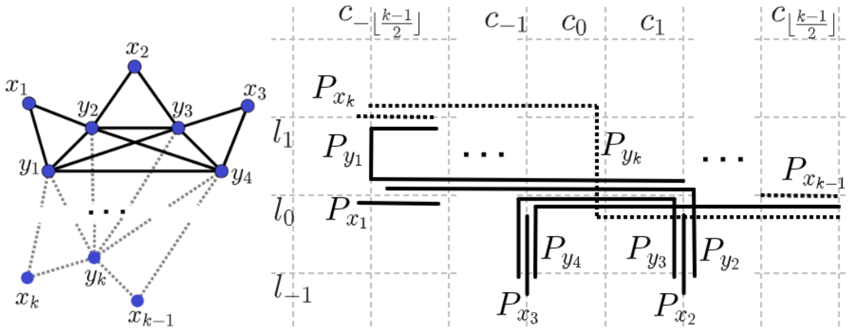
For the other paths P_{y_i} that represent the vertices of the clique Y (which exclude $i = 1$, $i = 2$, $i = k-1$ and $i = k$), do the following: For P_{y_i}, where i is odd, P_{y_i} has a horizontal segment on line l_0 between columns $c_{-\frac{i-1}{2}}$ and $c_{\frac{i-1}{2}}$, at both extremities P_{y_i} bends and extends to line l_{-1}. For P_{y_i}, where i is even, P_{y_i} has a horizontal segment on line l_0 between columns $c_{-(\frac{i}{2}-1)}$ and $c_{\frac{i}{2}}$, at both extremities P_{y_i} bends and extends to line l_{-1}.

The vertices of set X are segments distributed through the representation. For P_{x_i}, where $i = 1$, P_{x_i} is represented by a segment on line l_0 between columns $c_{-(\left\lfloor\frac{k-1}{2}\right\rfloor)}$ and $c_{-(\left\lfloor\frac{k-1}{2}\right\rfloor-1)}$. For P_{x_i}, where $i = k$, P_{x_i} is represented by a segment on line l_1 between columns $c_{-(\left\lfloor\frac{k-1}{2}\right\rfloor)}$ and $c_{-(\left\lfloor\frac{k-1}{2}\right\rfloor-1)}$. For the rest of paths P_{x_i}(which exclude $i = 1$, $i = k-1$ and $i = k$), do the following: For P_{x_i}, where i is even, P_{x_i} is represented by a vertical segment on column $c_{\frac{i}{2}}$ between lines l_0 and l_{-1}. for P_{x_i}, where i is odd, P_{x_i} is represented by a vertical segment on column $c_{-\frac{i-1}{2}}$ between lines l_0 and l_{-1}.

#

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

Figura 5 K-sun graph (at left) and a particular Helly B_2-EPG representation (at right).
To facilitate visualization, we present the Corollary 11 as a code in the Algorithm 2, in page 19



Concluding Remarks

This work presents a study of the B_2-EPG and Helly B_2-EPG classes. We present some general results about the B_2-EPG class approaching also geometric arrangements from paths and segments. In addition, we also present two demonstrations (by constructive algorithms) that prove the following results: k -sun graph ∈ B_2-EPG and k -sun graph ∈ Helly B_2-EPG. The next steps of this research are to investigate the behavior particular of some graph classes in B_2-EPG, e.g. chordal, split, and another. We would like to know about the Helly B_2-EPG graph recognizing problem. We agree that B_2-EPG graph recognizing problem is NP-Complete, because the related results in literature, e.g. B_1-EPG graph recognizing is NP-Complete, similarly also Helly B_1-EPG graph recognizing is NP-Complete. Maybe proving the monotonicity of the hierarchy within the B_k-EPG class is also an interesting field for future work.

References

Alcón, L., et al. (2018). On the Bend Number of Circular-arc Graphs as Edge Intersection Graphs of Paths on a Grid. *Discrete Applied Mathematics*, *234*, 12-21.

Alcón, L., Mazzoleni, M. P., & Santos, T. D. (2021). Relationship Among B1-EPG, VPT and EPT Graphs Classes. *Discussiones Mathematicae Graph Theory.*

Asinowski, A., & Suk, A. (2009). Edge Intersection Graphs of Systems of Paths on a Grid with a Bounded Number of Bends. *Discrete Applied Mathematics*, *157*(14), 3174-3180.

Cameron, K., Chaplick, S., & Hoàng, C. T. (2016). Edge Intersection Graphs of L-shaped Paths in Grids. *Discrete Applied Mathematics*, 210, 185-194.

Cela, E., & Gaar, E. (2019). Monotonic Representations of Outerplanar Graphs as EdgeIntersection Graphs of Paths on a Grid. arXiv preprint arXiv:1908.01981.

Francis, M. C., & Lahiri, A. (2016). VPG and EPG Bend-Numbers of Halin Graphs. *Discrete Applied Mathematics, 215*, 95-105.

e-ISSN nº 2447-4266
Palmas, v. 9, n. 1, p. 1-12 2023
http://dx.doi.org/10.20873/uft.2447-4266.2023v9n1a00en

Golumbic, M. C., Lipshteyn, M. & Stern, M. (2009). Edge Intersection Graphs of Single Bend Pathson a grid. *Networks: An International Journal*, *54*(3), 130-138.

Golumbic, M. C., Lipshteyn, M., & Stern, M. (2013). Single Bend Paths on a Grid Have Strong Helly number 4: errata atque emendationes ad "Edge Intersection Graphs of Single Bend Paths on a Grid". *Networks, 62*(2), 161-163.

Heldt, D., Knauer, K., & Ueckerdt, T. (2014). Edge-Intersection Graphs of Grid Paths: thebend number. *Discrete Applied Mathematics*, 167, 144-162.

Heldt, D., Knauer, K., & Ueckerdt, T. (2014). On the Bend-Number of Planar and Outerplanar graphs. *Discrete Applied Mathematics*, 179, (109-119).

Marinho, L. F. S., Silva, K. A., & Santos, T. D. B2-EPG Split. (2023). *Academic Journal on Computing, Engineering and Applied Mathematics, 4*(1), 1-7, 2023.

Pergel, M., & Rzążewski, P. (2017). On edge intersection graphs of paths with 2 bends. *Discrete Applied Mathematics*, 226, 106-116.

Stern, M., Biedl, T. (2010). On Edge-Intersection Graphs of k-Bend Paths in Grids. *Discrete Mathematics and Theoretical Computer Science, 12*(1), 1-12.

Szwarcfiter, J. L., et al. (2020). The Complexity of Helly-B_1-EPG Graph Recognition. *Discrete Mathematics and Theoretical Computer Science*, *22*.

RESUMO:
Neste artigo exploramos a classe de grafos B_2-EPG e a propriedade Helly. Apresentamos resultados genéricos sobre representações EPG e definimos termos que suportam os demais resultados, além disso, apresentamos um algoritmo inédito que constrói uma representação B_2-EPG-Helly de qualquer grafo k-sun.

PALAVRAS-CHAVE: B_2-EPG; Teoria dos Grafos; Grafos de Interseção; Propriedade Helly; Grafos *k*-sun.

RESUMEN:
En este artículo exploramos la clase B_2-EPG y la propiedad Helly. Presentamos resultados genéricos sobre las representaciones EPG y definimos términos que respaldan los otros resultados, además, finalizamos la investigación con un algoritmo original que construye una representación B_2-EPG -Helly de cualquier grafo *k*-sun.

PALABRAS CLAVE: B_2-EPG; Teoría de Grafos; Grafos de Intersección; Propiedad Helly; Grafos k-sun.